

Monolithic Semiconductor Chips as a Source for Broadband Wavelength-Multiplexed Polarization Entangled Photons

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Generating entangled photons from a monolithic chip is a major milestone towards real-life applications of optical quantum information processing including quantum key distribution and quantum computing. Ultrabroadband entangled photons are of particular interest to various applications such as quantum metrology and multi-party entanglement distribution. In this work, we demonstrate the direct generation of broadband wavelength multiplexed polarization entangled photons from a semiconductor chip for the first time. Without the use of any off-chip compensation, interferometry, entangled photons with a signal-idler separation as large as 95 nm in the telecom band were observed. The highest concurrence of 0.98 ± 0.01 achieved in this work is also the highest, to the best of our knowledge, comparing to all previously demonstrated semiconductor waveguide sources. This work paves the way for fully integrated, ultrabroadband sources of polarization entangled photons.

Entangled photons play pivotal roles in quantum information processing (QIP). They are essential sources for optical quantum computing (QC) [1], photon-based quantum metrology (QM) [2] and quantum key distribution (QKD) [3]. Each application calls for entangled photon pairs with specific attributes. Interest in broadband entangled photon pairs has increased in recent years. This has been motivated by various applications in quantum optical technologies such as quantum optical coherence tomography (QOCT) [4] and quantum optical frequency comb sources (QOFC) [5, 6]. In QOCT, the use of ultrabroadband photon pairs with extremely short correlation times can enhance the accuracy of protocols for quantum positioning and timing beyond the classical limits [4]. In the regime of continuous variable (CV) entanglement, QOFC, generated by placing a broadband spontaneous parametric down-conversion (SPDC) crystals in an optical parametric oscillator (OPO) cavity, is a crucial resource for multi-mode entanglement and cluster states [5]. Spectroscopy modalities that are enabled by entangled photon sources also rely on the availability of such photons within a broad range of energies to match transitions of various material systems [7].

One emerging application of entangled photons is in the multi-party entanglement distribution networks [8], which can empower multi-user QKD systems and quantum communication networks. In this case, broadband entangled photon pairs are required by the service provider who, by virtue of the relevant multi-user QKD protocols is required to generate multiple pairs of entangled photons at virtually the same time and subsequently distribute paired photons to multiple users at different locations. To realize such entanglement distribution networks, one practical methodology involves utilizing existing fiber communication links and dense wavelength-division-multiplexing (DWDM) systems to re-route entangled photon pairs that possess slightly different wave-

lengths to users at different locations. As such, the service provider for these systems requires the availability of a vast array of entangled photon sources at different wavelengths. Given the form-factor of existing single channel entangled photon sources, the scalability of these systems can be greatly enhanced if a single source can generate all the required entangled photon pairs [9]. There are no existing approaches or technology, which are able to address this unmet demand.

The generation of wavelength-multiplexed entangled photons requires a combination of techniques for producing both broad bandwidth and polarization entanglement. Typical photon pairs generated via type-II SPDC processes have a narrow bandwidth of only a few nanometers [10, 11], due to the material birefringence, and therefore are not suitable for this application because of the insufficient use of the fiber communication bandwidth. Broadband photon pairs generated via type-I or type-0 processes can be used to generate wavelength-multiplexed entangled photons, however, only with complex interferometric setups due to the lack of orthogonal polarizations from a single process [12–14]. For practical applications, compact, robust sources of entangled photons capable of integrating with other components in the system should be developed. Recent developments in integrated quantum photonics have achieved compact generation of entangled photons in monolithic waveguide devices using SPDC [15–17] or spontaneous four-wave mixing (SFWM) [18–20]. However, the demonstration of broadband wavelength-multiplexed entangled photons based on a single waveguide chip has not been reported. In addition, these existing semiconductor sources all suffer from low degree of entanglement.

In this work, we demonstrate a single-chip broadband wavelength multiplexed polarization entangled photon source for the first time. The source is a modal phase matched Bragg reflection waveguide (BRW) [16], con-

sisting of multiple layers of AlGaAs, in which the pump light is guided by Bragg reflections from two periodic reflectors while the down-converted photons are guided by conventional total internal reflections. Because AlGaAs is non-birefringent, cross-polarized photons generated from the type-II process have a much broader spectrum than those generated from birefringent crystals, allowing for wavelength multiplexing using standard DWDM. In addition, due to the lack of birefringence, the down-converted photons propagate at almost the same group velocities, which enables the generation of wavelength multiplexed polarization entangled photons directly from the chip without any off-chip compensation or interferometry. The degree of entanglement is also the highest to date among all semiconductor waveguide sources, to the best of our knowledge. Based on III-V semiconductors, this source allows monolithic integration of its own pump laser, which can lead to electrically pumped, room-temperature operating sources of broadband wavelength multiplexed polarization entangled photons.

RESULTS

Theory: Broadband entanglement in Bragg reflection waveguides

BRWs have been extensively studied in the past few years for classical wavelength conversions [21, 22] and entangled photon pair generations [16, 17, 23]. The two distinct guiding mechanisms involved provide a great design flexibility in achieving phase matching near the bandgap of semiconductors where formidable material dispersion exists, as well as in generating photon pairs with particulate quantum properties, such as entanglement [23–25] and spectral separability [26].

Of particular interest to this work, it was proposed that dispersion engineered BRWs with zero group velocity mismatch (GVM) between photons in a pair can generate ultra-broad band wavelength multiplexed polarization entangled photons [27]. However, even without any waveguide dispersion engineering which sacrifices the generation efficiency, cross-polarized photon pairs generated via a type-II SPDC process in a BRW showed inherent polarization entanglement without any off-chip compensation, usually required to remove the walk-off between photons in a pair, due to the lack of material birefringence [16]. As such, it can be expected that the photon pairs have a much broader bandwidth than a typical type-II process in birefringent materials because the bandwidth is largely determined by the GVM between photons in a pair.

We consider the same BRW demonstrated for polarization entanglement generation in [16], with the detailed structure described in Methods. For a 1.09 mm long

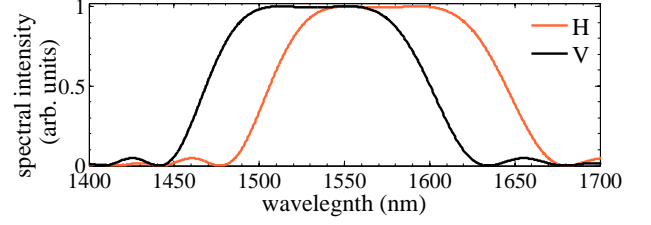


FIG. 1. The simulated spectra for the H and V polarized SPDC photons from a 1.09 mm long BRW using a CW pump.

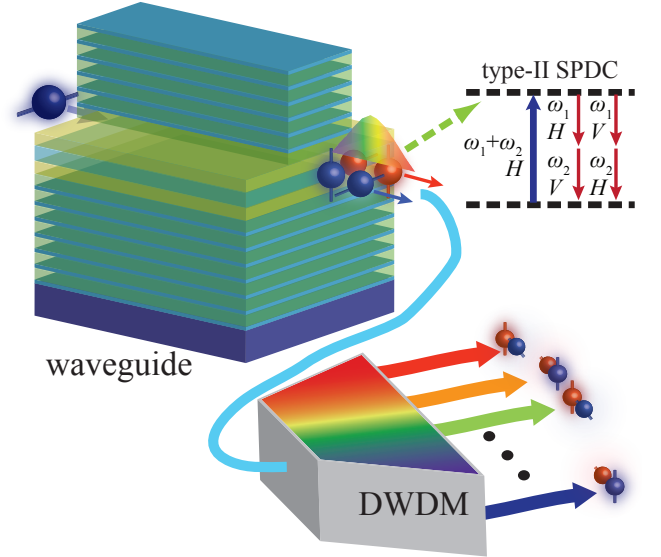


FIG. 2. The schematic setup of the thought experiment. Broadband polarization entangled photons are generated via a type-II SPDC process in the waveguide. They are subsequently coupled into an optic fiber and sent to a DWDM to be deterministically separated into many wavelength channel pairs.

waveguide in this work, the simulated spectra for the horizontal (H) and vertical (V) polarized down-converted photons using a CW pump are shown in Fig. 1. The spectrally overlapped region of the two polarizations has a FWHM of ~ 95 nm, which is considerably wider than that of a typical type-II process in a birefringence material.

Now consider the experiment that a DWDM filter is used after the waveguide to deterministically separate the down-converted photons into many wavelength channel pairs, as schematically shown in Fig. 2. The output two-photon state of each channel pair is given by

$$|\psi_n\rangle = \frac{1}{\sqrt{2}} \iint_{B_n} d\omega_s d\omega_i [\Phi_{HV}(\omega_s, \omega_i) a_H^\dagger(\omega_s) a_V^\dagger(\omega_i) + \Phi_{VH}(\omega_s, \omega_i) a_V^\dagger(\omega_s) a_H^\dagger(\omega_i)] |vac\rangle, \quad (1)$$

where ω_s, ω_i represent the signal, idler channel frequencies; $\Phi_{VH}(\omega_s, \omega_i), \Phi_{HV}(\omega_s, \omega_i)$ are the associated joint

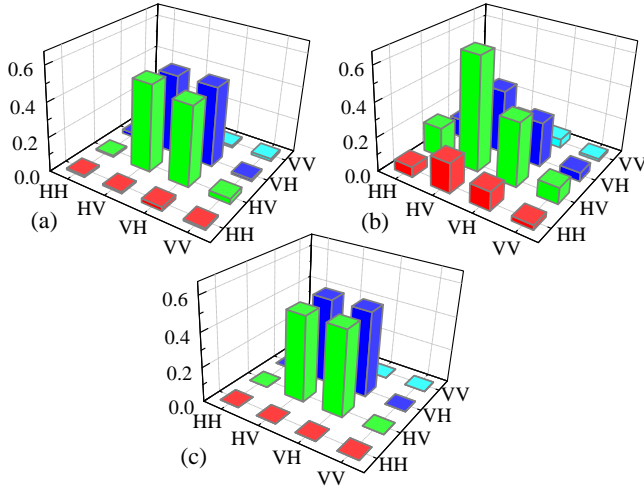


FIG. 6. The amplitudes of the reconstructed net density matrices for channel pairs of (a) 1554.0-1557.4 nm and (b) 1509.7-1604.6 nm, as well as (c) that of the maximally entangled state.

$(|HV\rangle + \exp(i\phi)|VH\rangle)/\sqrt{2}$ is shown in Fig. 6(c).

The density matrices near the degenerate wavelength, such as the one in Fig. 6(a), show high resemblances with that of a maximally entangled state in Fig. 6(c), indicating a high degree of entanglement. When the wavelengths are significantly far from degeneracy, such as in 6(b), the available photon pairs decrease, along with the decrease of entanglement.

The results for all measured channel pairs are summarized in Fig. 7, which shows the concurrence calculated from the measured density matrix as a function of the signal and idler wavelengths. A high value of concurrence of at least 0.96 ± 0.02 and as high as 0.98 ± 0.01 , as well as a highest fidelity of 0.97 to a maximally entangled state $(|HV\rangle + \exp(i\phi)|VH\rangle)/\sqrt{2}$ could be observed for the signal-idler pairs from 1535.4 nm to 1575.6 nm, which approximately covers the whole C-band (1530 nm-1565 nm). This corresponds to a total bandwidth of over 40 nm for the down-converted photons, and equivalently 26 pairs of wavelength channels if the channel spacing is 100 GHz. For signal and idler wavelengths further apart, the concurrence starts to decrease as well as the coincidence counts. Nevertheless, a concurrence of 0.77 ± 0.09 could still be measured for the channel pair of 1509.7-1604.6 nm, with a wavelength separation of 95 nm. Note in S- and L-bands, the increased error bars could be due to the decreased coincidence counts. The concurrences being lower than expected could also be because of the errors in estimating the waveguide dispersion properties, which determine the bandwidth of photon pairs.

The measured entanglement bandwidth extends from S- to L-bands, which can enable a high utilization of existing fiber optic communication infrastructures from a single source. We emphasize that polarization entangle-

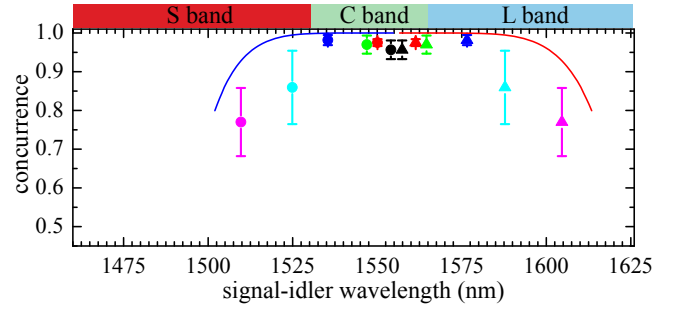


FIG. 7. The dependence of concurrence on the signal and idler wavelengths. Wavelength channels in the same pair are represented by the same color. The solid curves are the theoretical predictions from Fig. 3, with the degenerate wavelength shifted 6 nm to match that in the experiment.

TABLE I. Comparison of degree of entanglement generated from all demonstrated monolithic waveguide sources of polarization entangled photons.

Reference	Platform	Concurrence	Fidelity
[15]	AlGaAs	0.75 ± 0.05	0.87 ± 0.03
[16]	AlGaAs	0.52	0.83
[17]	AlGaAs	0.85 ± 0.07	0.89
[18]	silicon	0.88 ± 0.02	0.91 ± 0.02 ^a
[19]	silicon	NA	0.87
This work	AlGaAs	0.98 ± 0.01	0.97

^a The concurrence and fidelity in [18] are raw values without noise subtraction. As a comparison, the highest raw concurrence and fidelity obtained in this work are 0.92 ± 0.03 and 0.95, respectively. Results from all other references are noise subtracted.

ment is generated directly from the monolithic chip, without any off-chip compensation or interferometer, comparing with existing waveguide sources. The degree of entanglement is also the highest reported in all semiconductor waveguide sources of polarization entangled photons, as summarized in Table I.

Lastly, we show the quantum interference in a Bell-type experiment for a few channel pairs near the degenerate wavelength in H and D basis, in which the polarization of signal channel was set to be 0° and 45° , respectively, while the polarization of the idler channel was varied. Examples of interference patterns are given in Fig. 8(a) for channel pair 1550.9-1561.8 nm and Fig. 8(b) for channel pair 1547.0-1565.8 nm. The fitted curves, without accidental subtraction, show visibilities of 97.3% and 94.5% in the H basis, and 90.1% and 87.5% in the D basis, respectively.

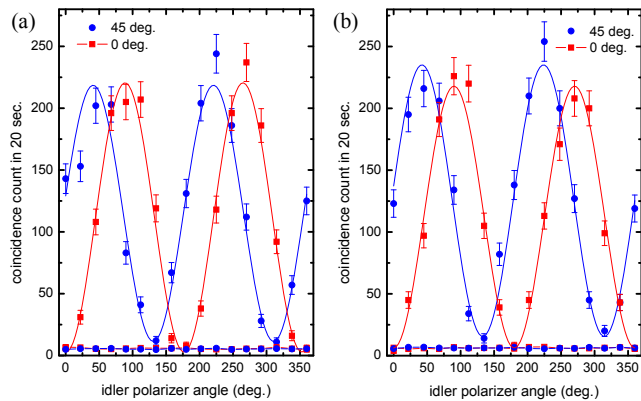


FIG. 8. Interference measurements for channel pairs (a) 1550.9-1561.8 nm and (b) 1547.0-1565.8 nm in the H and D basis. The solid curves are the sinusoidal fits. Accidental counts in each basis are also shown.

DISCUSSION

In summary, we demonstrated the generation of broadband wavelength multiplexed polarization entangled photons directly from a monolithic waveguide chip for the first time. With a minimal concurrence of 0.77, polarization entanglement was demonstrated in a bandwidth of 95 nm. In particular, for over 40 nm near the degenerate wavelength, high quality polarization entangled photons with a minimal concurrence of 0.96 and as high as 0.98 can be achieved. In addition, the degree of entanglement demonstrated in this work is the highest reported among all monolithic waveguide sources to the best of our knowledge.

The performance of the source can be improved by better device fabrications that decrease waveguide losses. As predicted in [32], the generation rate can be increased by orders of magnitude in a lossless waveguide. This will allow one to afford using a DWDM with a narrower channel bandwidth and closer channel separation. The degree of entanglement can be improved due to the increased spectral-temporal indistinguishability. On the other hand, the entanglement bandwidth could be increased via dispersion engineering, tapering, or by utilizing concurrent type-0 and type-I processes, as mentioned before.

Comparing with existing techniques for generating broadband wavelength multiplexed entangled photons, this method requires no interferometry, and thus greatly reduced the source complexity. Such a chip form-factor is essential for real life applications of entangled photons. In addition, based on a III-V semiconductor platform, we can envisage the integration of its own electrically pumped laser and passive components such as an array waveguide grating (AWG) on the same chip, which paves the way for fully integrated sources in quantum information processing and entanglement networks.

METHOD

Sample description

The semiconductor wafer was grown on a GaAs [001] substrate using metalorganic chemical vapor deposition (MOCVD). The composite core consists of a 500 nm thick $\text{Al}_{0.61}\text{Ga}_{0.39}\text{As}$ sandwiched by a pair of 375 nm $\text{Al}_{0.20}\text{Ga}_{0.80}\text{As}$. Each of the two Bragg mirrors consists of 6 pairs of 129/461 nm $\text{Al}_{0.25}\text{Ga}_{0.75}\text{As}/\text{Al}_{0.70}\text{Ga}_{0.30}\text{As}$. The waveguide used for polarization entanglement measurements has an etch depth of $3.6\text{ }\mu\text{m}$ and a ridge width of $4\text{ }\mu\text{m}$. The propagation losses for the down-converted photons were measured to be $\sim 4\text{ cm}^{-1}$.

Experimental details

The sample was mounted on a standard end-fire setup, with the pump from a cw Ti:sapphire laser set at TE polarization and the degenerate wavelength of 777.86 nm. A $100\times$ objective lens was used to focus the pump beam into the Bragg mode of the waveguide. After the sample, the output light was collected by a $40\times$ objective lens and filtered by a few long pass filters to reject the pump, and then coupled into a single mode fiber. A fiber based dichroic splitter was then subsequently used to deterministically separate paired photons into signal and idler paths according to wavelengths. One of the paths then subsequently passed through a tunable bandpass filter with a bandwidth set to be 55 GHz. This can select out paired photons in a given frequency channel pairs. Note that, ideally, an additional bandpass filter should be used in the other path to select the other twin photon in the same pair and suppress the uncorrelated noise photons. This could significantly reduce the accidental count rate while maintaining a high coincidence. Instead, a commercial DWDM could also be used to replace the filter system used here. Nevertheless, our filters emulates a DWDM in terms of the physics and allows us to explore the maximum bandwidth achievable for the device itself.

Photons in the two frequency channels were then launched into free-space for polarization manipulation and analyzing before being detected by a pair of InGaAs single photon detectors. The photons from the narrow frequency channel were detected by a free-running detector (id220, ID Quantique, 20% quantum efficiency), which triggered a gated detector (id210, ID Quantique, 25% quantum efficiency) used to detect photons from the wide frequency channel. An extra length of fiber was connected before the gated detector in order to compensate for the electronic delay between the two detectors. The detection events from both detectors were then sent to a time-to-amplitude converter (TAC) (id800, ID Quantique) with a timing resolution of $\sim 80\text{ ps}$ to record the coincidence histograms.

Near the degenerate wavelength, the peak net coincidence counts for all channels were approximately 350 counts in 20 seconds under an external pump power of 15 mW before the input objective lens. The pump power and integration time were chosen as a compromise of the coincidence to accidental (CAR) rate and the total measurement time, due to the mechanic instability of the experimental equipment. The peak CAR were ~ 50 in this case. Taking into account the pump coupling efficiency (7.3%), overall photon losses due to the output objective lens, fiber coupling, filters, wave-plates and polarizers, as well as detector efficiencies, we estimate a generation rate of 1×10^4 pairs/s/mW/GHz in terms output photon pairs after the waveguide and internal pump power. For two-photon interference measurements shown in Fig. 8, the external pump power was set to be 10 mW, when the stability of the setup can be sustained for a sufficiently long duration to carry out all the measurements.

To measure the polarization of each photon, a pair of birefringence compensators consisting of a half-wave plate (HWP) and a quarter-wave plate (QWP) was inserted in each path to partially compensate for the polarization rotation in the fiber, such that a given polarization before the detectors corresponds to the same polarization immediately after the waveguide. Projective polarization measurements were then performed using a QWP and a polarizer in each path.

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- [1] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, "Quantum computers," *Nature* **464**, 45 (2010).
- [2] V. Giovannetti, S. Lloyd, and L. Maccone, "Advances in quantum metrology," *Nature Photon.* **5**, 222 (2011).
- [3] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, "Quantum cryptography," *Rev. Mod. Phys.* **74**, 145 (2002).
- [4] M. B. Nasr, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, "Demonstration of Dispersion-Canceled Quantum-Optical Coherence Tomography," *Phys. Rev. Lett.* **91**, 083601 (2003).
- [5] M. Chen, N. C. Menicucci, and O. Pfister, "Experimental Realization of Multipartite Entanglement of 60 Modes of a Quantum Optical Frequency Comb," *Phys. Rev. Lett.* **112**, 120505 (2014).
- [6] J. Roslund, R. M. de Araújo, S. Jiang, C. Fabre, and N. Treps, "Wavelength-multiplexed quantum networks with ultrafast frequency combs," *Nature Photon.* **8**, 109 (2014).
- [7] F. Schlawin, K. E. Dorfman, B. P. Fingerhut, and S. Mukamel, "Suppression of population transport and control of exciton distributions by entangled photons," *Nature Comm.* **4**, 1782 (2013).
- [8] T. E. Chapuran, P. Toliver, N. A. Peters, J. Jackel, M. S. Goodman, R. J. Runser, S. R. McNown, N. Dallmann, R. J. Hughes, K. P. McCabe, J. E. Nordholt, C. G. Peterson, K. T. Tyagi, L. Mercer, and H. Dardy, "Optical networking for quantum key distribution and quantum communications," *New J. Phys.* **11**, 105001 (2009).
- [9] H. C. Lim, A. Yoshizawa, H. Tsuchida, K. Kikuchi, "Wavelength-multiplexed entanglement distribution," *Optical Fiber Technology* **16**, 225 (2010).
- [10] F. Kaiser, A. Issautier, L. A. Ngah, O. Dănilă, H. Herrmann, W. Sohler, A. Martin, and S. Tanzilli, "High-quality polarization entanglement state preparation and manipulation in standard telecommunication channels," *New J. Phys.* **14**, 085015 (2012).
- [11] H. Herrmann, X. Yang, A. Thomas, A. Poppe, W. Sohler, and Ch. Silberhorn, "Post-selection free, integrated optical source of non-degenerate, polarization entangled photon pairs," *Opt. Express* **21**, 27981 (2013).
- [12] H. C. Lim, A. Yoshizawa, H. Tsuchida, and K. Kikuchi, "Wavelength-multiplexed distribution of highly entangled photon-pairs over optical fiber," *Opt. Express* **16**, 22099 (2008).
- [13] I. Herbauts, B. Blauensteiner, A. Poppe, T. Jennewein, and H. Hübel, "Demonstration of active routing of entanglement in a multi-user network," *Opt. Express* **21**, 29013 (2013).
- [14] S. Arahira and H. Murai, "Nearly degenerate wavelength-multiplexed polarization entanglement by cascaded optical nonlinearities in a PPLN ridge waveguide device," *Opt. Express* **21**, 7841 (2013).
- [15] A. Orioux, A. Eckstein, A. Lemaître, P. Filloux, I. Favero, G. Leo, T. Coudreau, A. Keller, P. Milman, and S. Ducci, "Direct Bell States Generation on a III-V Semiconductor Chip at Room Temperature," *Phys. Rev. Lett.* **110**, 160502 (2013).
- [16] R. T. Horn, P. Kolenderski, D. Kang, P. Abolghasem, C. Scarcella, A. D. Frera, A. Tosi, L. G. Helt, S. V. Zhukovsky, J. E. Sipe, G. Weihs, A. S. Helmy, and T. Jennewein, "Inherent polarization entanglement generated from a monolithic semiconductor chip," *Sci. Rep.* **3**, 2314 (2013).
- [17] D. Kang, M. Kim, H. He, and Amr S. Helmy, "Two polarization-entangled sources from the same semiconductor chip," *Phys. Rev. A* **92**, 013821 (2015).
- [18] N. Matsuda, H. L. Jeannic, H. Fukuda, T. Tsuchizawa, W. J. Munro, K. Shimizu, K. Yamada, Y. Tokura, and H. Takesue, "A monolithically integrated polarization entangled photon pair source on a silicon chip," *Sci. Rep.* **2**, 817 (2012).
- [19] L. Olislager, J. Safioui, S. Clemmen, K. P. Huy, W. Bogaerts, R. Baets, P. Emplit, and S. Massar, "Silicon-on-insulator integrated source of polarization-entangled photons," *Opt. Lett.* **38**, 1960 (2013).

- [20] N. Lv, W. Zhang, Y. Guo, Q. Zhou, Y. Huang, and J. Peng, “1.5 μm polarization entanglement generation based on birefringence in silicon wire waveguides,” *Opt. Lett.* **38**, 2873 (2013).
- [21] P. Abolghasem, J.-B. Han, D. Kang, B. J. Bjilani, and A. S. Helmy, “Monolithic Photonics Using Second-Order Optical Nonlinearities in Multilayer-Core Bragg Reflection Waveguides,” *IEEE J. Sel. Top. Quantum Electron.* **18**, 812 (2012).
- [22] P. Abolghasem, D. Kang, D. F. Logan, M. Lungwitz, and A. S. Helmy, “Widely tunable frequency conversion in monolithic semiconductor waveguides at 2.4 μm ” *Opt. Lett.* **39**, 3591 (2014).
- [23] D. Kang and A. S. Helmy, “Generation of polarization entangled photons using concurrent type-I and type-0 processes in AlGaAs ridge waveguides,” *Opt. Lett.* **37**, 1481 (2012).
- [24] S. V. Zhukovsky, L. G. Helt, D. Kang, P. Abolghasem, A. S. Helmy, and J. E. Sipe, “Generation of maximally-polarization-entangled photons on a chip,” *Phys. Rev. A*, **85**, 013838 (2012).
- [25] D. Kang, L. G. Helt, S. V. Zhukovsky, J. P. Torres, J. E. Sipe, and A. S. Helmy, “Hyperentangled photon sources in semiconductor waveguides,” *Phys. Rev. A* **89**, 023833 (2014).
- [26] J. Svozilík, M. Hendrych, A. S. Helmy, and J. P. Torres, “Generation of paired photons in a quantum separable state in Bragg reflection waveguides,” *Opt. Express*, **19**, 3115 (2011).
- [27] J. Svozilík, M. Hendrych, and J. P. Torres, “Bragg reflection waveguide as a source of wavelength-multiplexed polarization-entangled photon pairs,” *Opt. Express* **20**, 15015 (2012).
- [28] W. K. Wootters, “Entanglement of Formation of an Arbitrary State of Two Qubits,” *Phys. Rev. Lett.* **80**, 2245 (1998).
- [29] F. Verstraete and M. M. Wolf, “Entanglement versus Bell Violations and Their Behavior under Local Filtering Operations,” *Phys. Rev. Lett.* **89**, 170401 (2002).
- [30] M. B. Nasr, S. Carrasco, B. E. A. Saleh, A. V. Sergienko, M. C. Teich, J. P. Torres, L. Torner, D. S. Hum, and M. M. Fejer, “Ultrabroadband Biphotons Generated via Chirped Quasi-Phase-Matched Optical Parametric Down-Conversion,” *Phys. Rev. Lett.* **100**, 183601 (2008).
- [31] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, “Measurement of qubits,” *Phys. Rev. A* **64**, 052312 (2001).
- [32] S. V. Zhukovsky, L. G. Helt, P. Abolghasem, D. Kang, J. E. Sipe, and A. S. Helmy, “Bragg reflection waveguides as integrated sources of entangled photon pairs,” *J. Opt. Soc. Am. B* **29**, 2516 (2012).